EE 232 Lightwave Devices
Lecture 3: Basic Semiconductor Physics and Optical Processes

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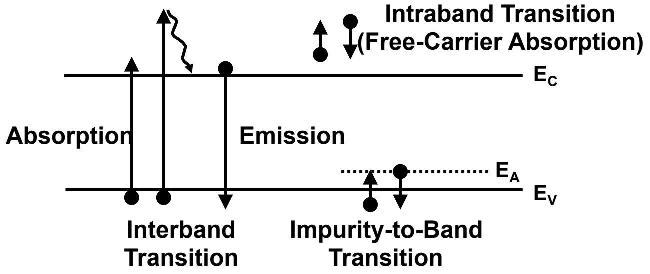
Optical Properties of Semiconductors



- Optical transitions
  - Absorption: exciting an electron to a higher energy level by absorbing a photon
  - Emission: electron relaxing to a lower energy state by emitting a photon

Band-to-Band Transition

- **Since most electrons and holes are near the band-edges, the photon energy of band-to-band (or interband) transition is approximately equal to the bandgap energy:**

$$h\nu = E_g$$

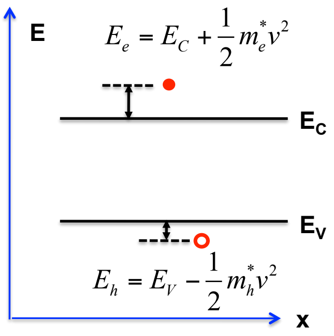
- **The optical wavelength of band-to-band transition can be approximated by**

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_g} \approx \frac{1.24}{E_g}$$

λ : wavelength in μm

E_g : energy bandgap in eV

Energy Band Diagram in Real Space and k-Space

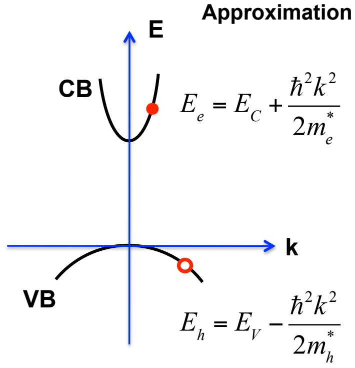


Real Space

Momentum:

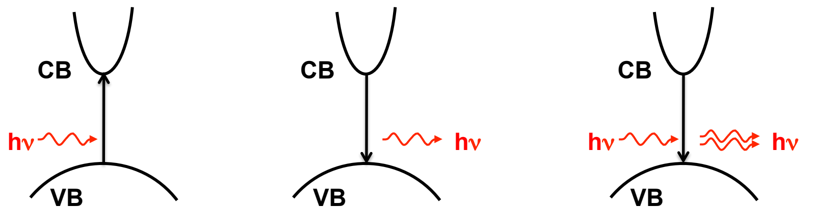
$$\hbar k = m_e^* v_e$$

Effective Mass Approximation



K-Space

Band-to-Band Transition

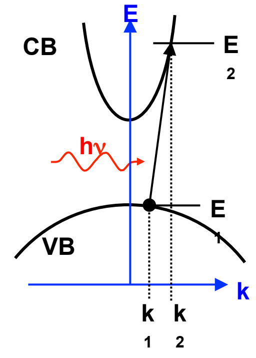


Absorption → **Photodetectors; Solar Cells**

Spontaneous Emission → **LED**

Stimulated Emission → **Optical Amplifiers; Semiconductor Lasers**

Conservation of Energy and Momentum



Optical transitions are "vertical" lines

- Conditions for optical absorption and emission:
  - Conservation of energy

$$\boxed{E_2 - E_1 = h\nu}$$

  - Conservation of momentum

$$k_2 - k_1 = k_{h\nu}$$

$$k_2, k_1 \sim \frac{2\pi}{a}$$

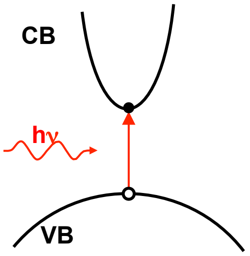
$$k_{h\nu} \sim \frac{2\pi}{\lambda}$$

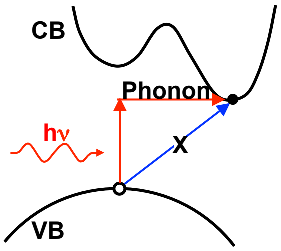
$$(a \sim 0.5nm) << (\lambda \sim 1\mu m)$$

Lattice Constant

$$\Rightarrow \boxed{k_2 = k_1}$$

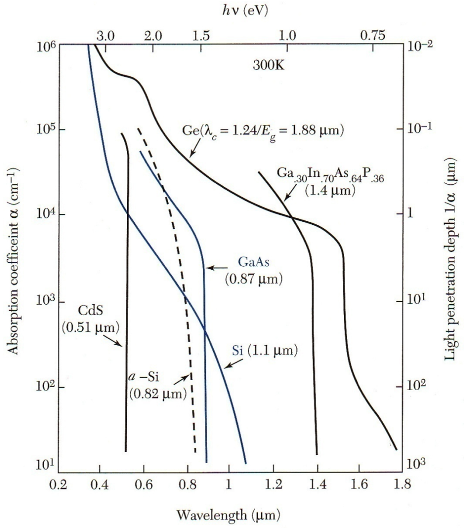
Direct vs Indirect Bandgaps





- Direct bandgap materials
  - CB minimum and VB maximum occur at the same k
  - Examples
    - GaAs, InP, InGaAsP
    - $(Al_xGa_{1-x})As$, $x < 0.45$
- Indirect bandgap materials
  - CB minimum and VB maximum occur at different k
  - Example
    - Si, Ge
    - $(Al_xGa_{1-x})As$, $x > 0.45$
  - Not "optically active"

Absorption Coefficient

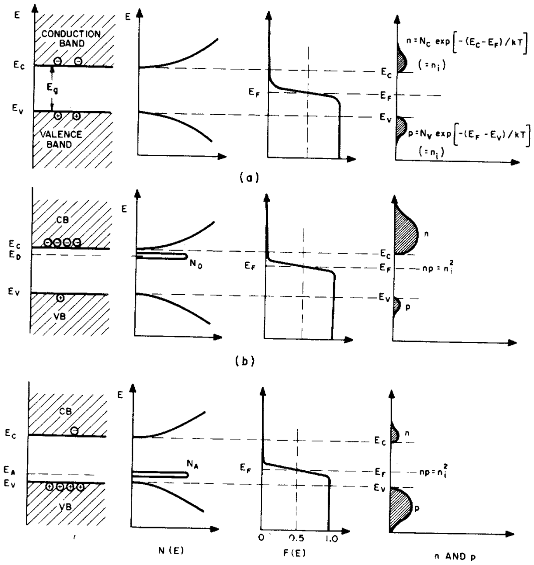


- Light intensity decays exponentially in semiconductor:

$$I(x) = I_0 e^{-\alpha x}$$

- Direct bandgap semiconductor has a sharp absorption edge
- Si absorbs photons with hv > E_g = 1.1 eV, but the absorption coefficient is small
  - Sufficient for CCD
- At higher energy (~ 3 eV), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB

Review of Semiconductor Physics



Electron and hole concentrations:

$$n = \int_{E_C}^{\infty} f_n(E)\rho_e(E)dE$$

$$p = \int_{-\infty}^{E_V} f_p(E)\rho_h(E)dE$$

Fermi-Dirac distributions:

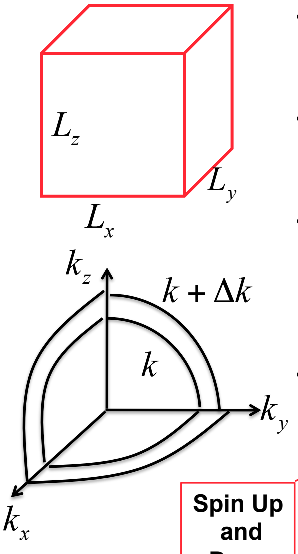
$$f_n(E) = \frac{1}{1+\exp\left(\frac{E-F_n}{k_B T}\right)}$$

$$f_p(E) = \frac{1}{1+\exp\left(\frac{F_p-E}{k_B T}\right)}$$

F_n : electron quasi-Fermi level

F_p : hole quasi-Fermi level

Electron/Hole Density of States (1)



- **Electron wave with wavevector k**

$$e^{i\vec{k}\cdot\vec{r}}$$

- **Periodic boundary conditions**

$$e^{i\vec{k}\cdot\vec{r}} = e^{i\vec{k}\cdot\left(\vec{r}+L_x\vec{x}\right)} = e^{i\vec{k}\cdot\left(\vec{r}+L_y\vec{y}\right)} = e^{i\vec{k}\cdot\left(\vec{r}+L_z\vec{z}\right)}$$

- **An electron state is defined by**

$$\left(k_x, k_y, k_z\right)\uparrow\downarrow = \left(m\frac{2\pi}{L_x}, n\frac{2\pi}{L_y}, l\frac{2\pi}{L_z}\right)\uparrow\downarrow$$

- **Number of electron states between k and k + Δk in k-space per unit volume**

$$\frac{2}{V}\cdot\frac{4\pi k^2 dk}{\frac{2\pi}{L_x}\frac{2\pi}{L_y}\frac{2\pi}{L_z}} = \frac{k^2}{\pi^2}dk = \rho_k(k)dk$$

Spin Up and Down

Electron/Hole Density of States (2)

- **Number of electron states between E and E + ΔE per unit volume**

$$E = E_C + \frac{\hbar^2 k^2}{2m_e^*} \Rightarrow dE = \frac{\hbar^2 k}{m_e^*} dk$$

$$\frac{k^2}{\pi^2} dk = \frac{m_e^*}{\hbar^2 \pi^2} \frac{\sqrt{2m_e^*\left(E - E_C\right)}}{\hbar} dE = \rho_e(E) dE$$

$$\rho_e(E) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \sqrt{E - E_C}$$

- **Likewise, hole density of states**

$$\rho_h(E) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} \sqrt{E_V - E}$$

Electron and Hole Concentrations

$$n = \int_{E_C}^{\infty} f_n(E) \rho_e(E) dE = \int_{E_C}^{\infty} \frac{1}{1 + \exp\left(\frac{E - F_n}{k_B T}\right)} \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2}\right)^2 \sqrt{E - E_C}\, dE$$

$$\boxed{n = N_C \cdot F_{1/2}\left(\frac{F_n - E_C}{k_B T}\right)}$$

$$N_C = 2\left(\frac{\pi m_e^* k_B T}{2\pi^2 \hbar^2}\right)^{3/2}$$

$$\boxed{\text{p}=N_V \cdot F_{1/2}\left(\frac{E_V - F_p}{k_B T}\right)}$$

$$N_V = 2\left(\frac{\pi m_h^* k_B T}{2\pi^2 \hbar^2}\right)^{3/2}$$

Fermi-Dirac Integral

$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^{\infty} \frac{x^j}{1 + e^{x-\eta}} dx$$

Gamma Function

$$\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$$

Approximation of Electron/Hole Concentration

$$F_j(\eta) = \frac{1}{\Gamma(j+1)}\int_0^\infty \frac{x^j}{1+e^{x-\eta}}dx \approx \begin{cases} e^\eta \text{ when } \eta << 1 \\ \frac{4}{3}\left(\frac{\eta^3}{\pi}\right)^{1/2} \text{ when } \eta >> 1 \end{cases}$$

When $F_n << E_C$ (Boltzmann approximati

$$n \approx N_C \cdot e^{-\frac{E_C - F_n}{k_B T}}$$

When $F_n >> E_C$ (Degenerate)

$$n \approx N_C \cdot \frac{4\left(\frac{F_n - E_C}{k_B T}\right)^{3/2}}{3\sqrt{\pi}}$$